



# Cambridge International AS & A Level

CANDIDATE  
NAME

--

CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--



**FURTHER MATHEMATICS**

**9231/22**

Paper 2 Further Pure Mathematics 2

**May/June 2020**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.





2 It is given that  $y = 2^x$ .

(a) By differentiating  $\ln y$  with respect to  $x$ , show that  $\frac{dy}{dx} = 2^x \ln 2$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Write down  $\frac{d^2y}{dx^2}$ . [1]

.....

.....

.....

(c) Hence find the first three terms in the Maclaurin's series for  $2^x$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

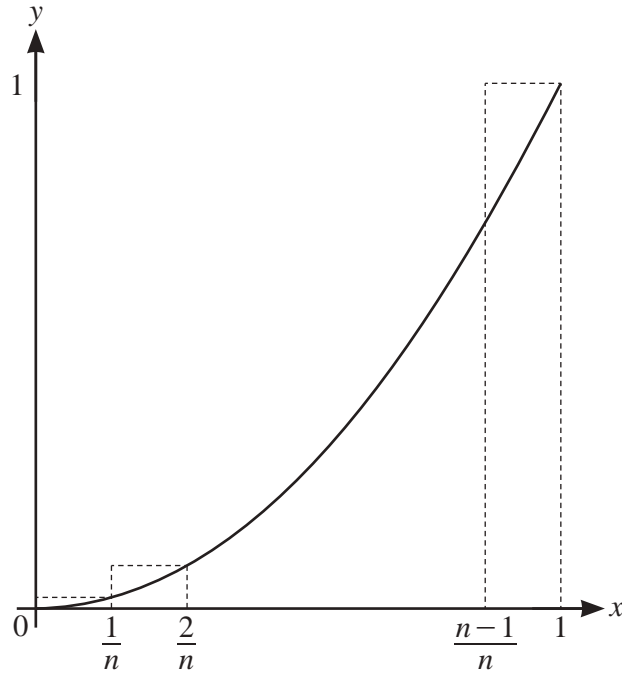
- 3 (a) Find the roots of the equation  $z^3 = -1 - i$ , giving your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $0 \leq \theta < 2\pi$ . [5]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

Let  $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$ , where  $k$  is a positive integer and  $z_1, z_2, z_3$  are the roots of  $z^3 = -1 - i$ .

- (b) Express  $w$  in the form  $Re^{i\alpha}$ , where  $R > 0$ , giving  $R$  and  $\alpha$  in terms of  $k$ . [3]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....



The diagram shows the curve with equation  $y = x^2$  for  $0 \leq x \leq 1$ , together with a set of  $n$  rectangles of width  $\frac{1}{n}$ .

(a) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \quad [4]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....  
.....  
.....  
.....  
.....  
.....  
.....

(b) Use a similar method to find, in terms of  $n$ , a lower bound for  $\int_0^1 x^2 dx$ . [4]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

5 The curves  $C_1 : y = \cosh x$  and  $C_2 : y = \sinh 2x$  intersect at the point where  $x = a$ .

(a) Find the exact value of  $a$ , giving your answer in logarithmic form. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Sketch  $C_1$  and  $C_2$  on the same diagram. [2]





6 The integral  $I_n$ , where  $n$  is an integer, is defined by  $I_n = \int_0^{\frac{1}{2}} (1-x^2)^{-\frac{1}{2}n} dx$ .

(a) Find the exact value of  $I_1$ . [2]

.....  
.....  
.....  
.....  
.....

(b) By considering  $\frac{d}{dx} \left( x(1-x^2)^{-\frac{1}{2}n} \right)$ , or otherwise, show that

$$nI_{n+2} = 2^{n-1} 3^{-\frac{1}{2}n} + (n-1)I_n. \quad [5]$$

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

- (c) Find the exact value of  $I_5$  giving the answer in the form  $k\sqrt{3}$ , where  $k$  is a rational number to be determined. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 It is given that  $x = t^3y$  and

$$t^3 \frac{d^2y}{dt^2} + (4t^3 + 6t^2) \frac{dy}{dt} + (13t^3 + 12t^2 + 6t)y = 6te^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 6te^{\frac{1}{2}t}. \quad [4]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the general solution for  $y$  in terms of  $t$ .

[7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 (a) Find the values of  $a$  for which the system of equations

$$3x + y + z = 0,$$

$$ax + 6y - z = 0,$$

$$ay - 2z = 0,$$

does not have a unique solution.

[3]

.....  
.....  
.....  
.....  
.....

The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

(b) Use the characteristic equation of  $\mathbf{A}$  to find the inverse of  $\mathbf{A}^2$ .

[4]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

(c) Find a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ . [7]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

